Estimating Failure Rates in the Absence of Failures

Frequently, when data are reviewed to develop component MTBF values for reliability, availability, and reliability (RAM) or life-cycle cost analyses, a number of components will have not exhibited any failures during the given operating period T. However, it is possible to make a statistical observation concerning the mean time between failure (MTBF) with a degree of confidence on the basis of the evidence that the equipment had operated during a period of time without failure. In general, a confidence bound of C% that the true MTBF is greater than a $MTBF_L$ (L = Lower Limit) having observed N failures in T hours of operation is given by:

$$MTBF_L = \frac{2T}{X^2_{100-C,N+2}}$$

Where: X^2 represents the Chi-square distribution for C% confidence and N+2 degrees of freedom.

Thus, an estimate of the lower confidence bound for a 95% confidence interval of a MTBF can be obtained from the data available using the following:

$$MTBF_L = \frac{2T}{X^2_{.05,2}}$$

This means that there is a 95% probability that the true, but unknown, MTBF is greater than .3338T. However if this lower bound is used as the MTBF input to a RAM analysis, unrealistically low estimates of unit performance will be obtained. Similarly, an upper 95% confidence bound can be defined in the same way as:

$$MTBF_U = \frac{2T}{X^2_{.95,2}}$$

This means that there is a 5% chance that the true, but unknown, MTBF is greater than 19.417T. This means that there is a 90% chance that the true, but unknown MTBF is somewhere between .3338T and 19.417T. However, having these two values does not provide an estimate of the *mean* value of the MTBF within the interval. To determine the value should be used for the component MTBF in the no failure case, given that it is possible to estimate the 90% bounds, it is suggested that the 50% confidence bound be used to estimate the MTBF sample mean. This would be saying, in effect, that there is a 50% chance that the true value of the mean is greater than this value and there is a 50% chance that it is less than this value. Thus, the estimate of the MTBF will be given by:

$$MTBF = \frac{2T}{X^2_{.5,2}}$$

Hence, for a time period 3.18 years:

$$MTBF = \frac{2*T}{X_{50,2}} = \frac{2*3.2}{1.386} = 4.618\, years$$

This approach can also be used when there is only 1 failure in time T, which what was assumed for this study. This approach is recommended because, although an estimate of the MTBF can be obtained (i.e. MTBF = T), the variance of this estimate is infinite and bounds cannot be established. In this case, the three input values would be given by:

$$MTBF_L = \frac{2T}{X^2_{.05,3}}$$

$$MTBF = \frac{2T}{X^2_{.5,3}})$$

$$MTBF_U = \frac{2T}{X^2_{.95,3}}$$